Quantitative Spectroscopy: From Radiative Transfer Theory to Stellar Atmosphere Codes

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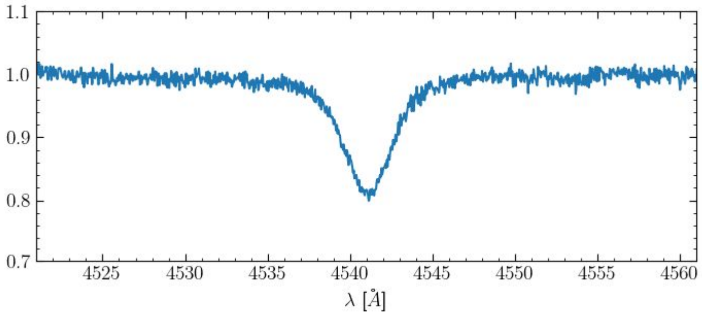
Massive Stars Lecture Series 4

23th of April 2026

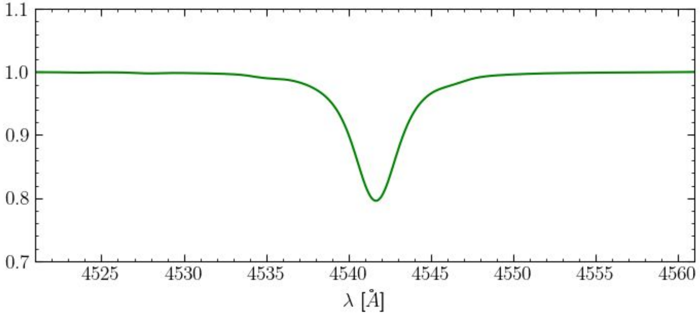
Quantitative Spectroscopy of Massive Stars

Block 4: Quantitative Spectroscopy of Massive Stars

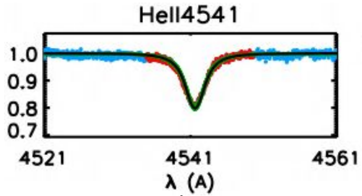
Observed spectrum



Synthetic spectra



Comparison



Parameters

Teff (kK)	=	41.5 +/− 1.6
log g (dex)	=	3.90 +/− 0.13
Y(He) x 10^2	=	9.8 +/− 2.5
ξ_t (kms^{-1})	<	9.0 +/+ 12.3
log $\mathcal{L}/\mathcal{L}_\odot$	=	3.96 +/− 0.10
−log Q	=	−12.69 +/− 0.75
β	=	1.00 +/− 0.00
R ($R_\odot$)	=	10.4 +/− 0.3
log ($L/L_\odot$)	=	5.45 +/− 0.05
M ($M_\odot$)	=	32.2 +/− 8.0
−log Mdot	=	0.00 +/− 0.00

Block 4: Quantitative Spectroscopy of Massive Stars

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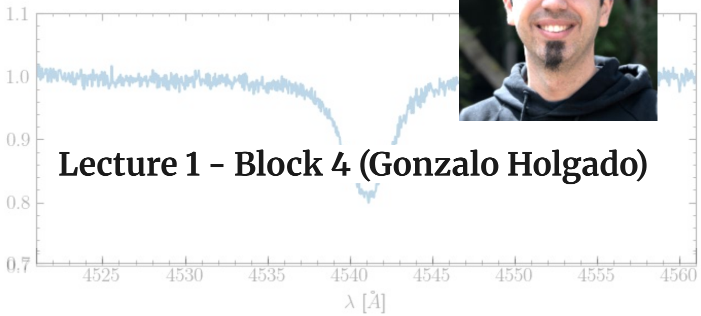
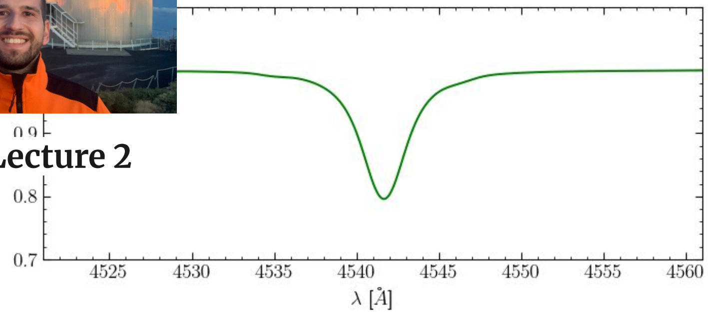


Lecture 1 – Block 4 (Gonzalo Holgado)

Synthetic spectra



Lecture 2

Comparison

Lecture 3 – Block 4 (Evgeny Nikolaeva)

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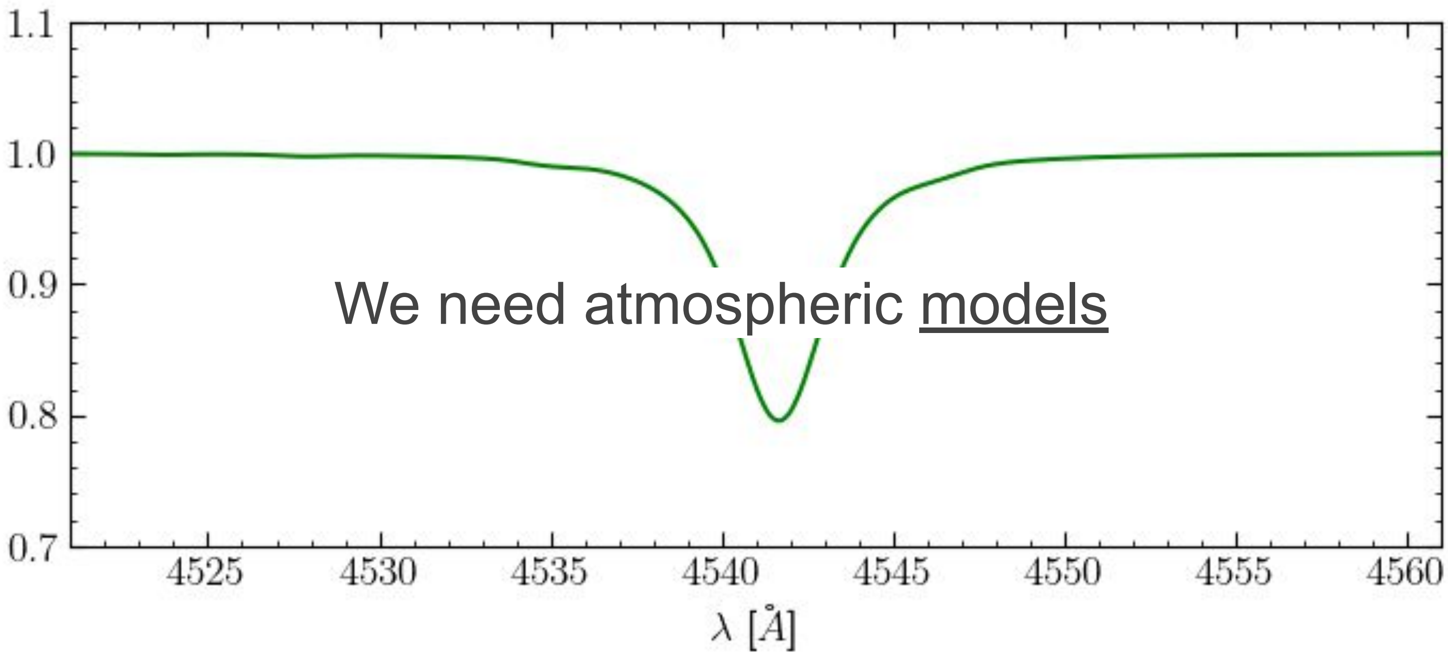
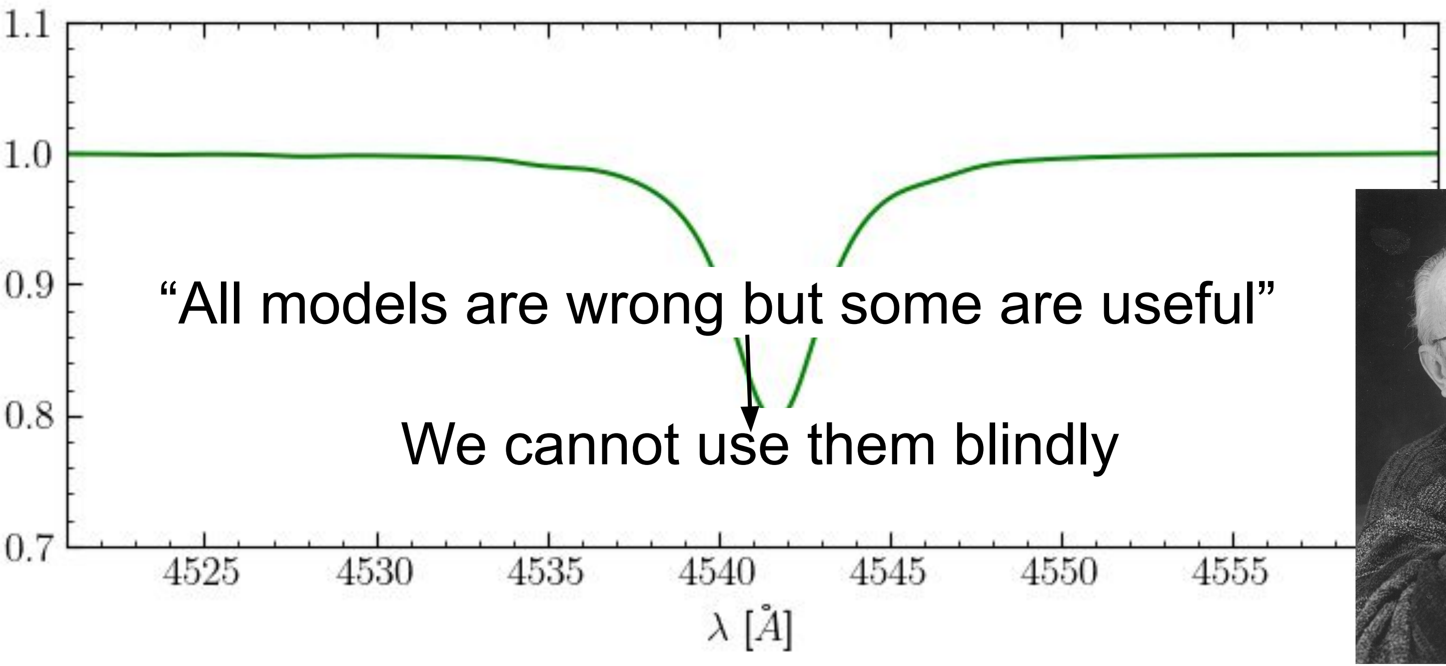
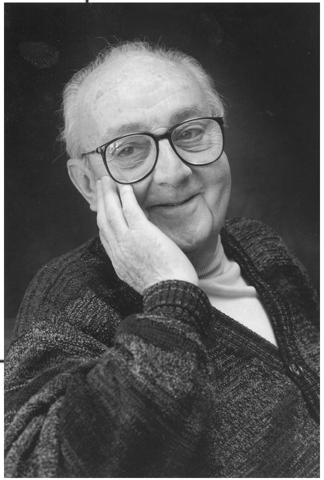


We need atmospheric models

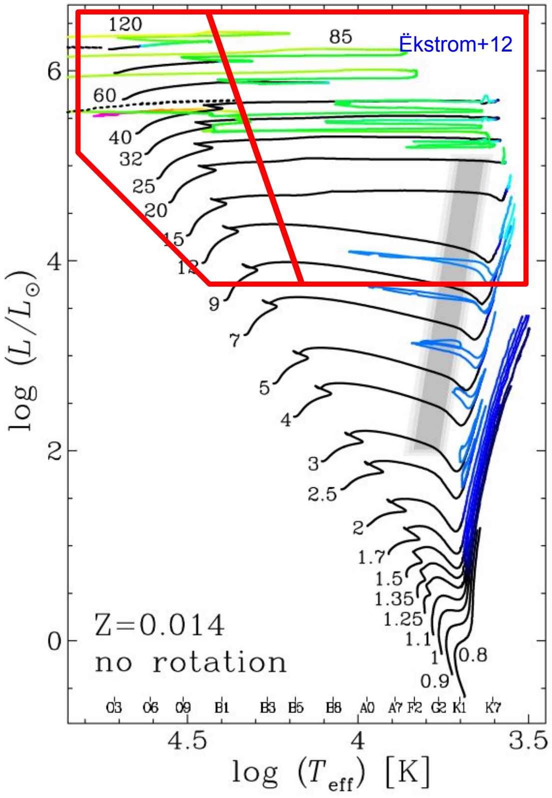


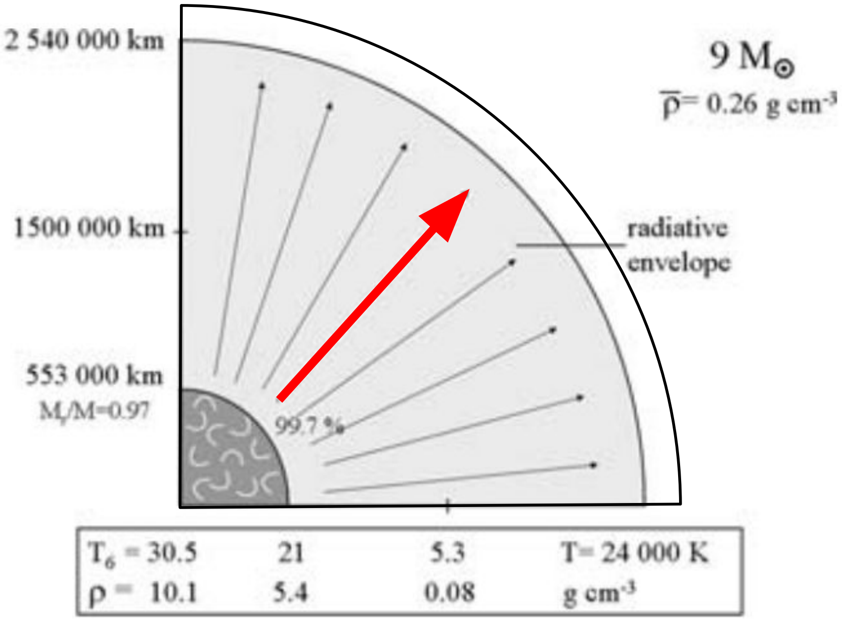
“All models are wrong but some are useful”

We cannot use them blindly



George E. P. Box

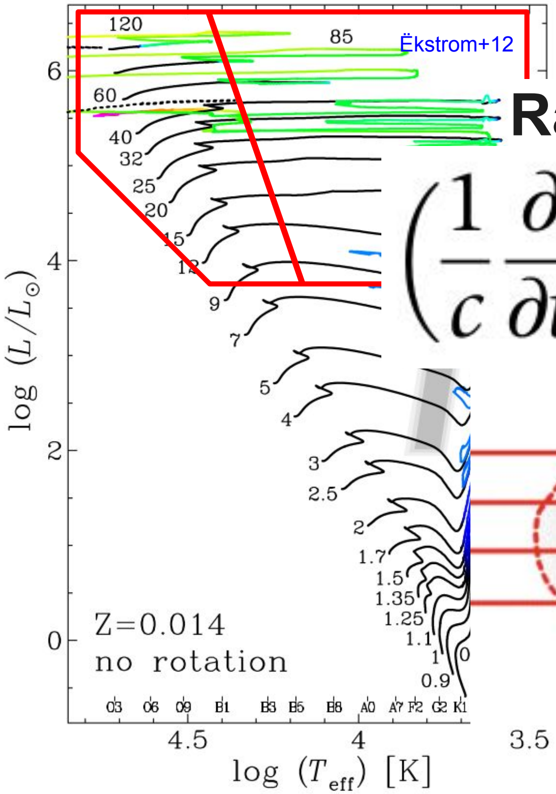




- Low density
- Intense radiation
- Intense wind

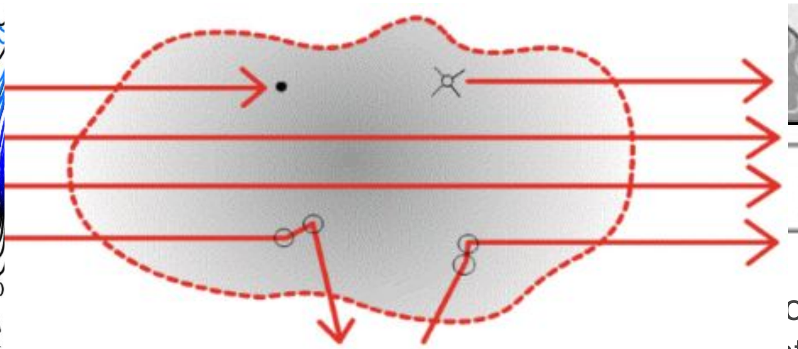
Lec. 2, B. 1: Massive star - Stars that can undergo a core collapse
O, B, BSg, YSg, RSg, LBV, WR

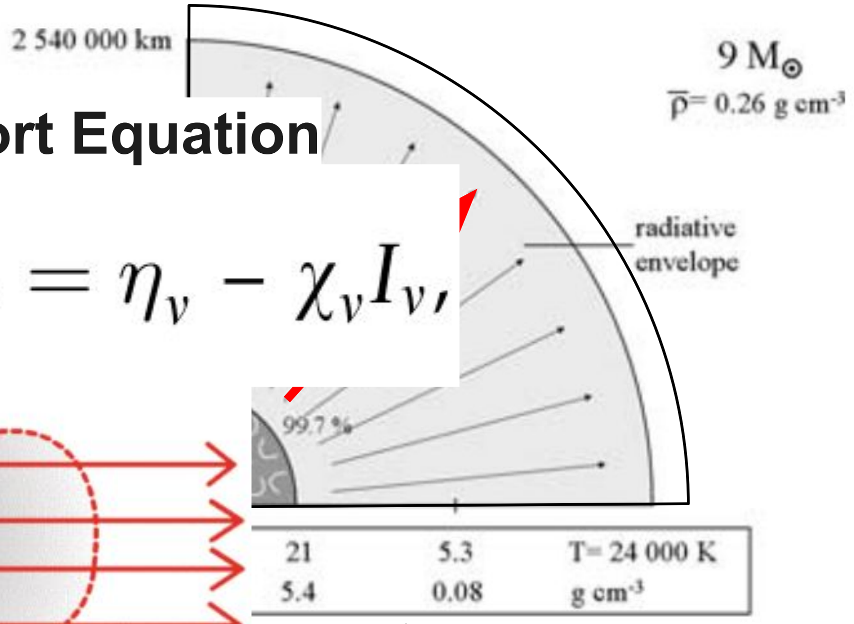
Unified models



Radiative Transport Equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}\cdot\nabla\right) I_\nu = \eta_\nu - \chi_\nu I_\nu,$$





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Unified models

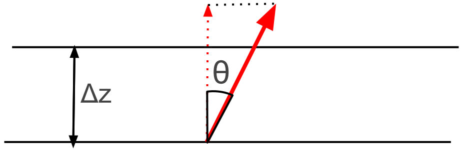
Stellar atmosphere physics

1D models

Plane-parallel

$\Delta z << R_*$

$\mu = \cos\theta$



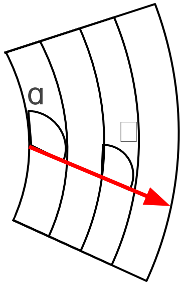
High gravities + low density winds

If stationary:

$$\mu \frac{\mathrm{d}}{\mathrm{d}z} I_\nu(z,\mu) = \eta_\nu - \chi_\nu I_\nu,$$

Spherically symmetric

Extended atmospheres

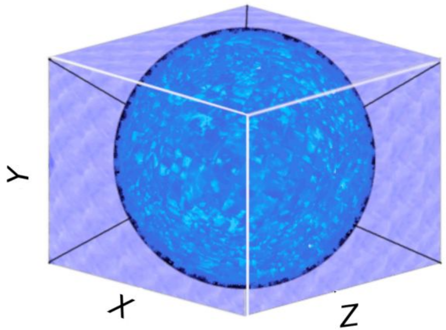


If stationary:

$$\left(\mu \frac{\partial}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu}\right) I_\nu(r,\mu) = \eta_\nu - \chi_\nu I_\nu,$$

3D models

Hydrodynamic simulations to investigate inhomogeneities + rotation + polarization



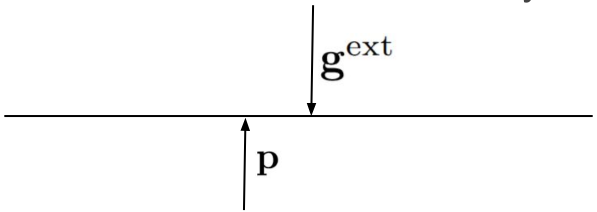
Delbroek et al. (2025)

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}\cdot\nabla\right) I_\nu = \eta_\nu - \chi_\nu I_\nu,$$

Structure equations

Atmospheric plasma behaves as an ideal gas – $p = Nk_\mathrm{B}T$

Maxwellian velocity distribution



Hydrodynamic equations:

- Continuity: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

- Momentum (Euler equation): $$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v}) = -\nabla p + \rho \boldsymbol{g}^{\mathrm{ext}},$$

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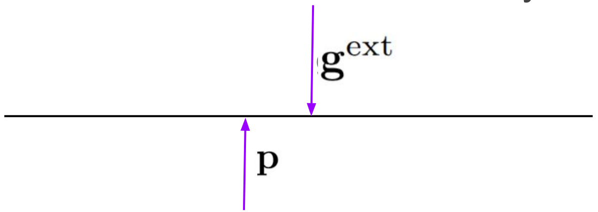

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No velocity → Hydrostatic equilibrium → $\mathrm{d}p/\mathrm{d}z = \rho g^{\mathrm{ext}},$

$$\rho(z) = \rho(z_0) \exp\left(-\frac{(z - z_0)}{H}\right)$$

Scale height: $H = \frac{k_{\mathrm{B}} T_{\mathrm{phot}}}{\mu m_{\mathrm{H}} g_*} = 2 \frac{v_{\mathrm{s}}^2}{v_{\mathrm{esc}}^2} R_*, \quad g_* = \frac{G M_*}{{R_*}^2},$

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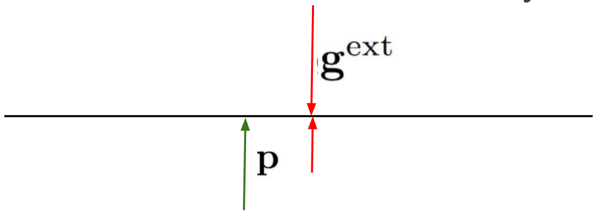

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Stationary velocity field
Spherically symmetric

$$\rho v(r) \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p}{\mathrm{d}r} + \rho g^{\text{ext}}(r),$$

$$r^2 \rho v(r) = \text{const} = \frac{\dot{M}}{4\pi},$$

Stellar wind

See lectures of Block 2

$$\left(1 - \frac{v_{\mathrm{s}}^2}{v^2}\right) \rho v(r) \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{GM}{r^2} + g^{\mathrm{o}}(r) + \frac{2v_{\mathrm{s}}^2}{r} - \frac{\mathrm{d}v_{\mathrm{s}}^2}{\mathrm{d}r}$$

with $p = \rho v_{\mathrm{s}}^2,$

Stellar winds

Pressure driven winds

$$\left(1-\frac{v_s^2}{v^2}\right)\rho v(r)\frac{dv}{dr}=-\frac{GM}{r^2}+g^o(r)+\frac{2v_s^2}{r}-\frac{dv_s^2}{dr}$$

Initiated by high-temperature corona

Solar type stars

Irrelevant for the evolution

Continuum driven winds

$$\left(1-\frac{v_s^2}{v^2}\right)\rho v(r)\frac{dv}{dr}=-\frac{GM}{r^2}+g^o(r)+\frac{2v_s^2}{r}-\frac{dv_s^2}{dr}$$

Theoretically relevant for very metal poor stars

Thomson scattering in porous medium

Dust Driven winds

$$\left(1-\frac{v_s^2}{v^2}\right)\rho v(r)\frac{dv}{dr}=-\frac{GM}{r^2}+g^o(r)+\frac{2v_s^2}{r}-\frac{dv_s^2}{dr}$$

Important for cool stars

Photons accelerate dust, + viscous drag force

Line-driven winds

$$\left(1-\frac{v_s^2}{v^2}\right)\rho v(r)\frac{dv}{dr}=-\frac{GM}{r^2}+g^o(r)+\frac{2v_s^2}{r}-\frac{dv_s^2}{dr}$$

Large oscillator strength of line transitions (compared to continuum cross sections) + high luminosity

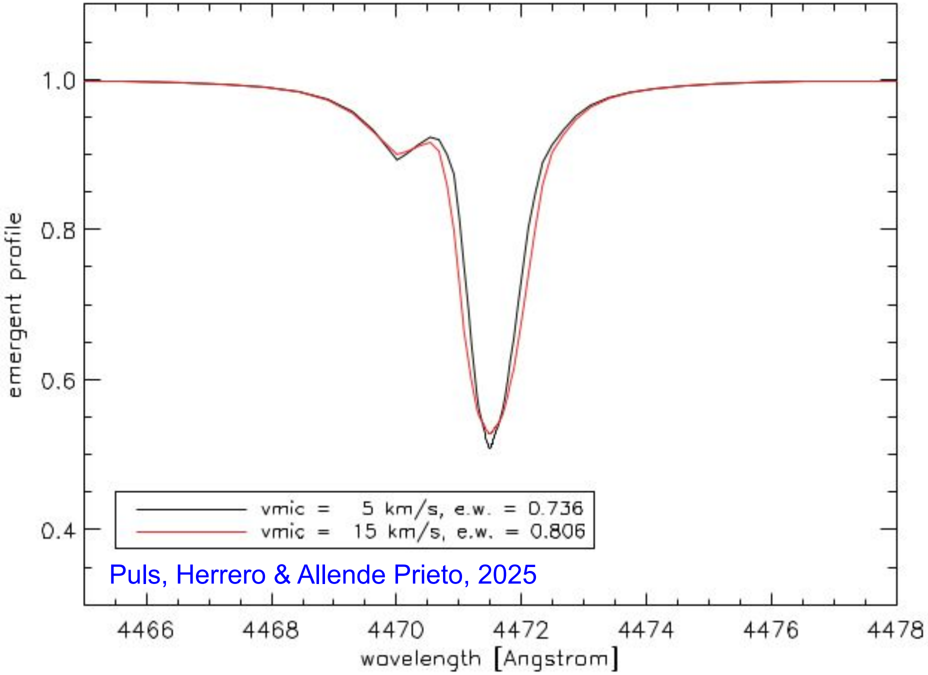
↓

large photon momentum transferred by line scattering and absorptions

Peak of the flux: where the atoms can absorbe the photons

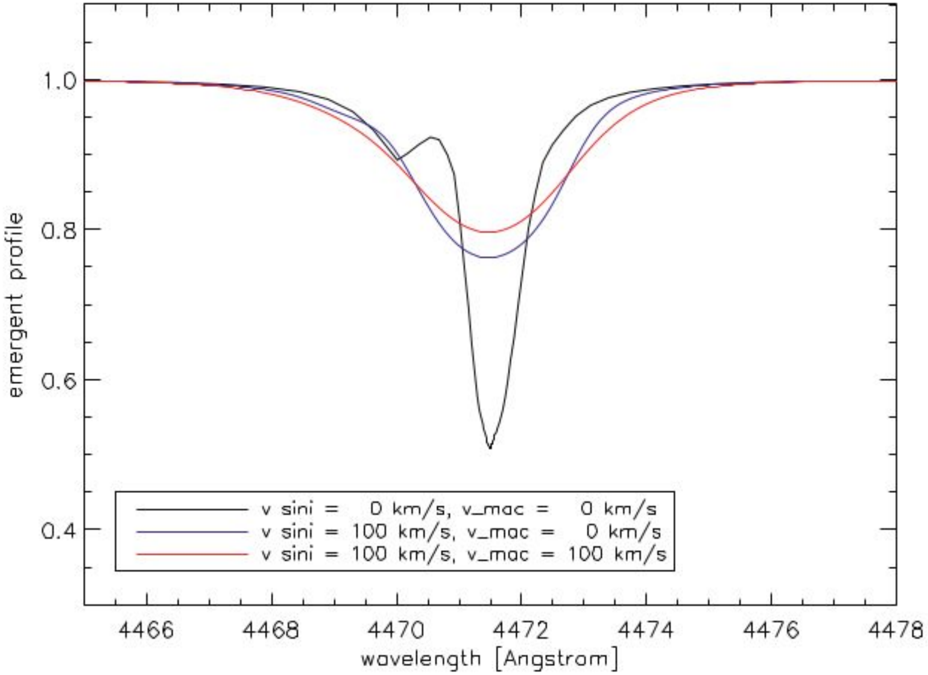
Inhomogeneities (clumping)
Usually treated as optically thin
Approximated treatment for optically thick at specific wavelengths

Microturbulence



Puls, Herrero & Allende Prieto, 2025

Macroturbulence



Energy conservation

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n} \cdot \nabla\right) I_\nu = \eta_\nu - \chi_\nu I_\nu,$$

In stationary & static atmospheres:

Flux conservation: $$\nabla \cdot [\mathcal{F} + \boldsymbol{F}^{\mathrm{c}} + \boldsymbol{F}^{\mathrm{conv}} + \ldots] = 0,$$

In **radiative atmospheres**, assuming only radiative energy transport:

Radiative cooling

Radiative equilibrium: $$\nabla \cdot \mathcal{F} = \int_0^\infty \mathrm{d}\nu \oint \mathrm{d}\Omega (\eta_\nu - \chi_\nu I_\nu) = \Lambda - \mathcal{H} \stackrel{!}{=} 0$$

Radiative heating

Energy conservation

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Radiative heating

$$L = 4\pi r^2 \mathcal{F}(r) = 4\pi R_*{}^2 \mathcal{F}(R_*) = 4\pi R_*{}^2 \sigma_{\mathrm{B}} T_{\mathrm{eff}}^4 = \mathrm{const.}$$

Used to derive the temperature structure

Rosseland mean opacity
Thermalization
large τ + p-p

$$T^4(\tau_{\mathrm{R}}) \approx \frac{3}{4} T_{\mathrm{eff}}^4 \left(\tau_{\mathrm{R}} + \frac{2}{3}\right)$$

Exact solution → Iterative process

Opacities and emissivities

RTE: $$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}\cdot\nabla\right)I_\nu = \eta_\nu - \chi_\nu I_\nu,$$

Bound-bound transitions

Free-free processes: Einstein–Milne relations

Bound-free transitions: Einstein–Milne relations

Thomson scattering:

$$\chi_\nu^{\mathrm{Th}} = \sigma_{\mathrm{Th}} n_{\mathrm{e}} \qquad \text{with} \qquad \sigma_{\mathrm{Th}} = \frac{8\pi}{3} r_0^2 \approx 6.65 \cdot 10^{-29}\,\mathrm{m}^2$$

$$\eta_\nu^{\mathrm{Th}} \approx \chi_\nu^{\mathrm{Th}} J_\nu$$ Needs iterative process

Rayleigh scattering

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Rayleigh scattering

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Opacity: $\chi_\nu^{\mathrm{line}} \approx \frac{h\nu_{\mathrm{ul}}}{4\pi}\left[n_{\mathrm{l}} B_{\mathrm{lu}} \phi(\nu) - n_{\mathrm{u}} B_{\mathrm{ul}} \psi(\nu)\right]$

Emissivity: $\eta_\nu^{\mathrm{line}} = \frac{h\nu_{\mathrm{ul}}}{4\pi} \psi(\nu) n_{\mathrm{u}} A_{\mathrm{ul}}$

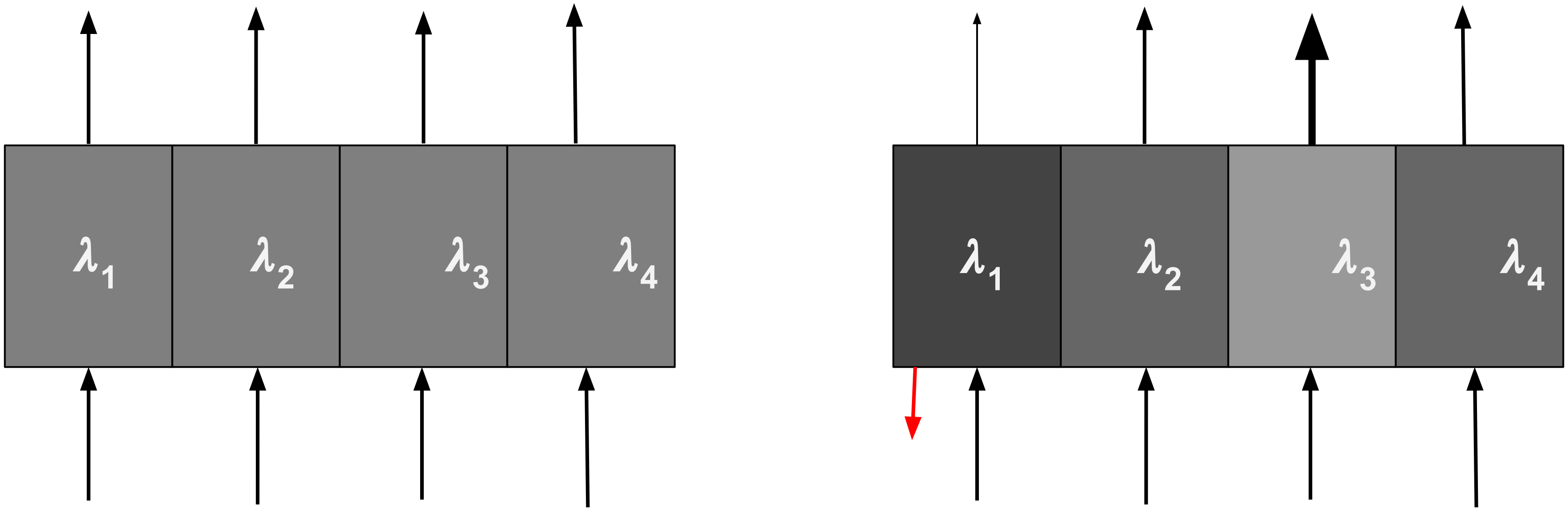
$B_{\mathrm{lu}}, B_{\mathrm{ul}}, A_{\mathrm{ul}}$: Einstein coefficients for absorption, induced and spontaneous emission

$\phi(\nu), \psi(\nu)$: Absorption & emission profile function

Complete redistribution: We consider $\psi(\nu) \rightarrow \phi(\nu)$

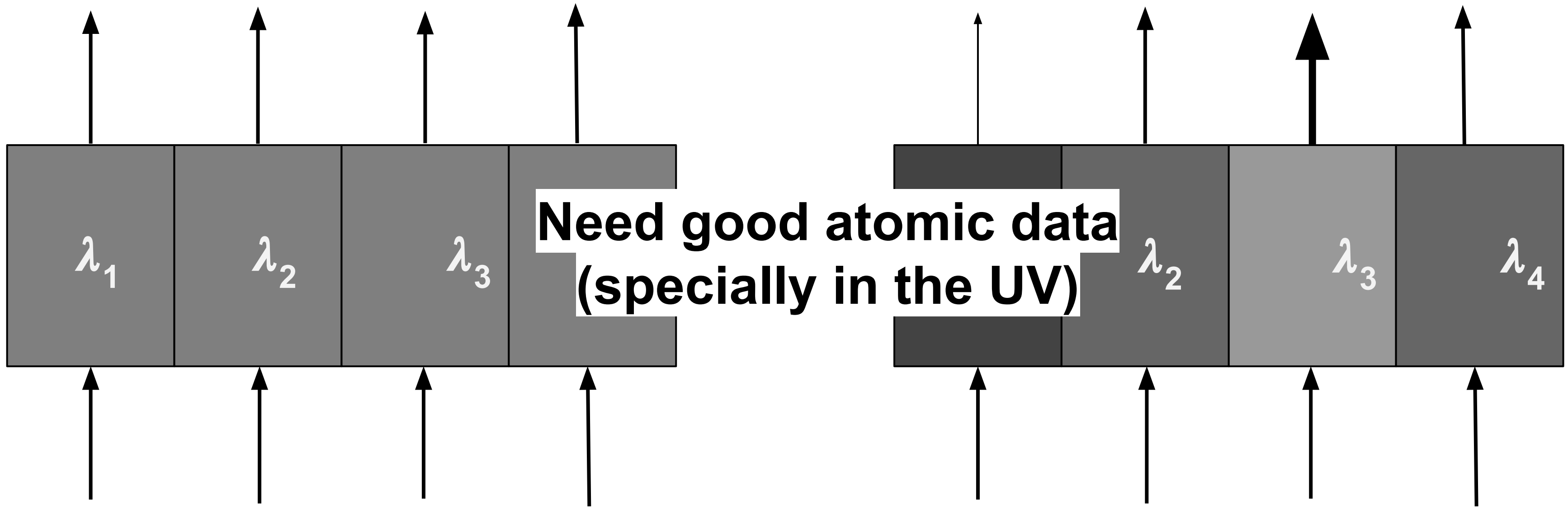
n: Occupation number density

Intermezzo – Line blanketing



Flux redistribution + Back-warming (and surface-cooling)

Included with
superlevels, opacity distribution function, opacity sampling…



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Atomic physics: population equilibrium

If **collisions dominates plasma**: Local thermodynamic equilibrium (LTE)

Laws of thermodynamic equilibrium are valid at the local temperature

Occupation numbers: Saha-Boltzmann equation $$\frac{N_{jk}}{N_{j+1,k}} = 2.07\times10^{-16} N_e \frac{U_{jk}}{U_{j+1,k}} T^{-\frac{3}{2}} e^{\chi_{jk}/kT}$$

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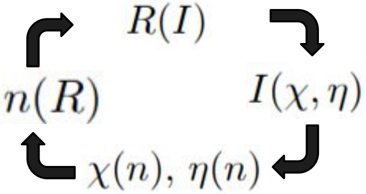
In **other cases**: statistical equilibrium or **NLTE**

$$\sum_{j \neq i} n_i P_{ij} = \sum_{j \neq i} n_j P_{ji} \longrightarrow n_i \sum_{j \neq i} (R_{ij} + C_{ij}) = \sum_{j \neq i} n_j (R_{ji} + C_{ji})$$



Occupation number and radiation field have to be computed in parallel

Problems with
Physics: much more atomic data than in LTE

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Lecture 4 - Block 4
(Alba Casasbuenas)

$$R(I) \qquad n(R) \qquad I(\chi, \eta) \qquad \chi(n),\ \eta(n)$$

Occupation number and radiation field have to be computed in parallel

Atomic physics: population equilibrium

Problems with
Physics: much more atomic data than in LTE
Computation: Need for methods which converge to the correct solution

In **other cases**: statistical equilibrium or **NLTE**

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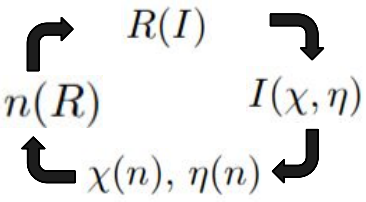


Occupation number and radiation field have to be computed in parallel

Stellar atmosphere codes

Stellar atmosphere codes

Table 1: Comparison of state-of-the-art, NLTE, line-blanketed model atmosphere codes.

code	Detail/ Surface[1]	TLUSTY[2]	POWR[3]	PHOENIX[4]	CMFGEN[5]	WM-basic[6]	FASTWIND[7]
geometry	plane-parallel	plane-parallel	spherical	spherical/ pl.-parallel	spherical	spherical	spherical
blanketing	LTE	yes	yes	yes	yes	yes	approx.
diagnostic range	no limitations	no limitations	no limitations	no limitations	no limitations	UV	optical/IR
major application	BA stars with negl. winds	hot stars with negl. winds	WRs	cool stars, SNe	OB(A)-stars, WRs, SNe	hot stars w. dense winds, SNe	OB-stars, early A-sgs
comments	no wind	no wind	–	no clumping no X-rays	–	no clumping	no X-rays
execution time	few minutes	hours	hours	hours	hours	1 to 2 h	few min. to 0.5 h

(1) Giddings (1981), Butler & Giddings (1985); (2) Hubeny (1998), (3) Gräfener et al. (2002), (4) Hauschildt (1992), (5) Hillier & Miller (1998), (6) Pauldrach et al. (2001), (7) Puls et al. (2005)

From Puls (2009)

Codes: FASTWIND

Santolaya-Rey et al. (1997), Puls et al. (2005)

- Spherically symmetric, non-LTE, electron thermal balance
- Not significantly optically thick wind in the (optical) continuum
- It can treat also optically thick clumping (approximate way)
- Explicit elements (CMF radiative transfer line transition)
- Background elements (mostly Sobolev approximation – Not important line)
- Blanketing - Approximate (Puls et al., 2005)
- Wind strength parameter for recombination lines:

$$Q_{\rm ws} = \frac{\dot{M}\sqrt{f_{\rm cl}}}{M_\odot\,{\rm yr}^{-1}} \left(\frac{{\rm km\,s}^{-1}}{v_\infty}\frac{R_\odot}{R_{2/3}}\right)^{3/2}$$

- Execution time: ~ minutes

Codes: CMFGEN

Hillier & Miller (1998)

- Spherically symmetric, non-LTE, radiative equilibrium
- Unified model. For winds of all strength (including optically thick)
- Radiative transfer in CMF
- Line-blanketing - Superlevel approach
- Time-dependent branch for SNe

- Execution time: ~ (many) hours

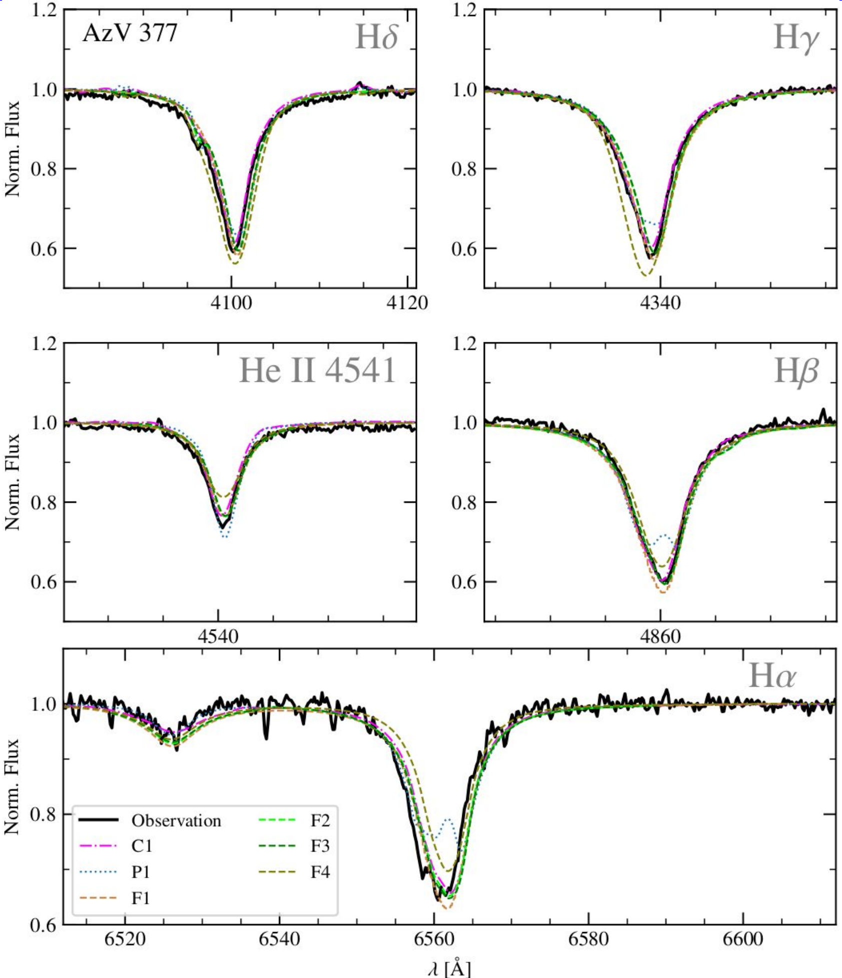
Codes: PoWR

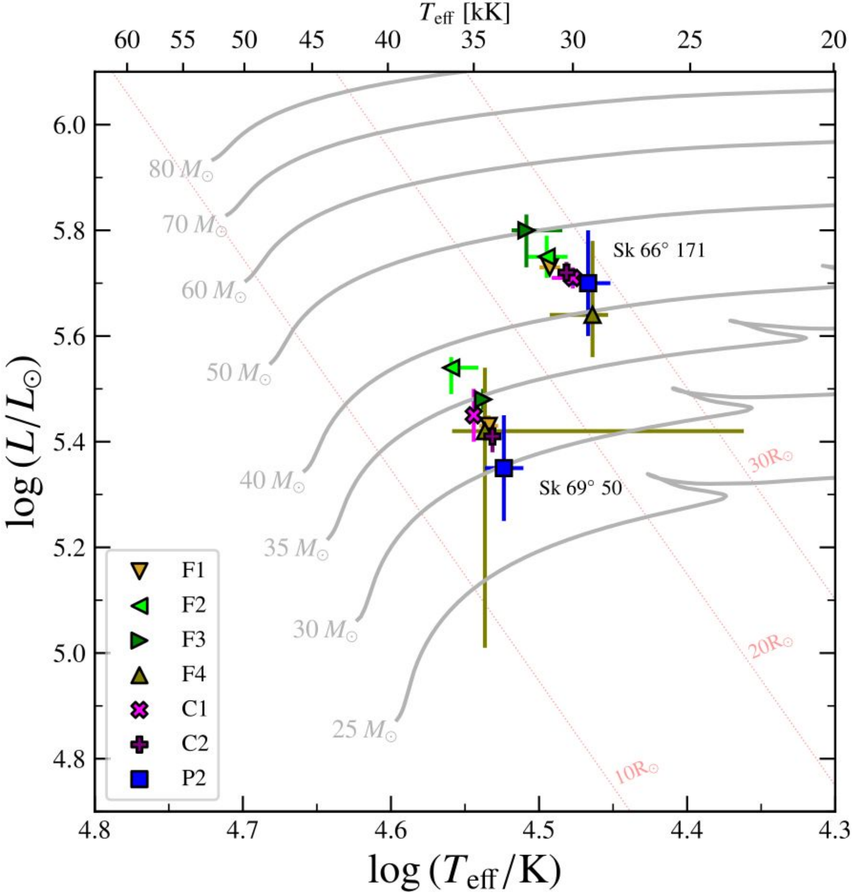
Gräfener et al. (2002), Hamann & Gräfener (2003)

- Spherically symmetric, non-LTE, radiative equilibrium
- Similar parameter space as CMFGEN - Different numerical approach
- T_{eff} + R - Output parameters
- Line-blanketing - Superlevel approach
- By default, turbulent pressure is included in the hydrostatic equation → Difference in the derived gravity

- Execution time: ~ hours to many hours (depending on number of superlevels per ion)

Codes: Some comparisons





From Sanders et al. (2024)

Codes: Some conclusions

Different methods scatter

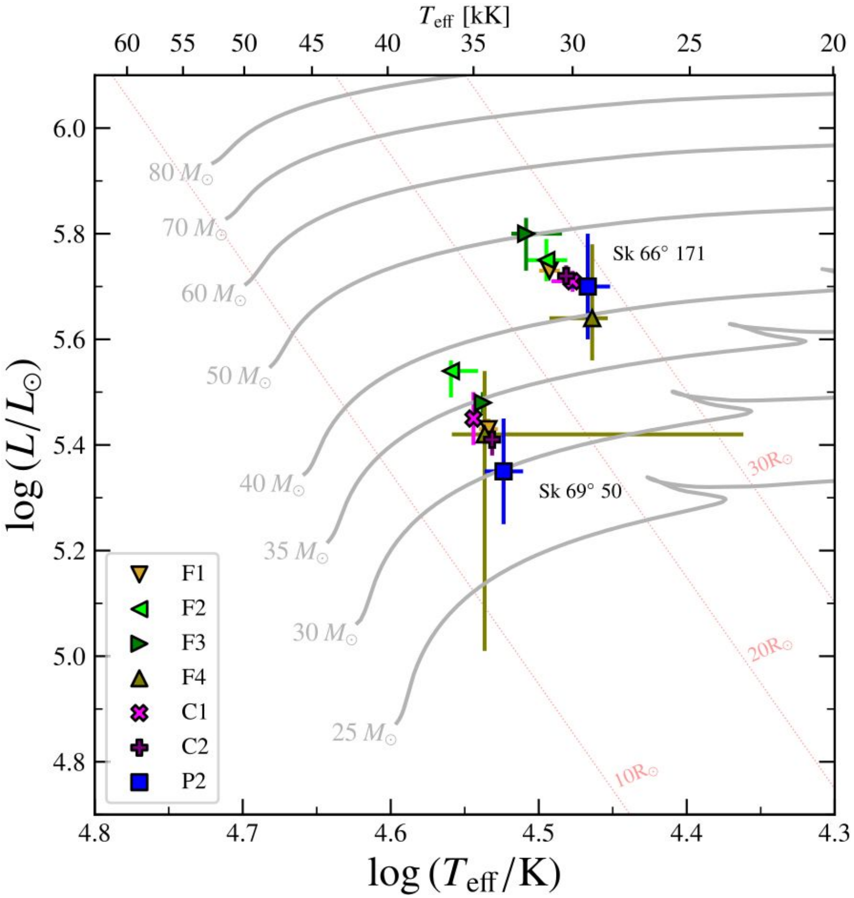
Heterogeneous choices – Big impact due to the treatment of the clumping and fitting lines

Impact of the atomic data

"[...] the scatter obtained in this work should reflect the amount of scatter to be expected when "blindly" combining data from different literature sources."
Sanders et al. (2024)

Which code should I use?

It depends on your science, but keep track of your assumptions and approaches



From Sanders et al. (2024)

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General references

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